

Soft Theorems from String Theory

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Soft behaviour of closed string amplitudes involving dilatons, gravitons and anti-symmetric tensors, is studied in the framework of bosonic string theory. The leading double soft limit of gluons is analysed as well, starting from scattering amplitudes computed in the open bosonic string. Field theory expressions are then obtained by sending the string tension to infinity. The presented results have been derived in the papers of Ref [1].

1 Introduction

There is a long history of studying the relations between the soft behaviour of field theory amplitudes and the symmetries of the underlying theory.

Since the 50s it has been shown that the leading behaviour of scattering amplitudes with a soft photon is obtained by means of gauge invariance from the corresponding amplitudes without the soft particle [2]. The extension of this theorem to the universal leading behaviour of amplitudes with one soft graviton was discussed by Weinberg in the 60s [3].

The nonlinear realization of symmetries provides another example of the relation existing between symmetries and low energy theorems. When a group G is spontaneously broken to some subgroup H , Nambu-Goldstone bosons appear and parametrize the coset space G/H . Their interaction with the other fields charged under the symmetry group is described in the Lagrangian by derivative couplings. As a result, amplitudes involving one soft Nambu-Goldstone boson are vanishing [4, 5]. These are the famous Adler's zero studied for the first time in the contest of pion dynamics [6].

Recently the leading divergent behaviour of amplitudes with a soft graviton was obtained from the Ward identity [7] of the diagonal Bondi, van der Burg, Metzner and Sachs supertranslation symmetry [8]. Later, similar results have been obtained for Yang-Mills theory where the soft gluon theorem arises as the Ward identity of a two dimensional Kac-Moody type symmetry [9]. The ex-

tension of these theorems to subleading order for gluons, and sub-subleading order for gravitons, have been obtained by computing on-shell scattering amplitudes [10, 11] and proved in arbitrary dimensions by using Poincaré and on-shell gauge invariance [12, 13]. In these new soft theorems, $n + 1$ -point amplitudes with a soft graviton or gluon are obtained acting on n -point hard amplitudes with universal soft operators depending on the momenta and polarizations of the hard particles.

The symmetries of the quantum field theory are also reflected in the double soft behaviour of scattering amplitudes with scalar particles or gluons. This has been made explicit in Ref. [4] where, in the case of spontaneously broken symmetries, it has been shown that amplitudes with two soft Nambu-Goldstone bosons capture the algebra of the broken generators of the global symmetry.

More recently supergravity amplitudes involving fermion particles have been studied in three and four dimensions and in the kinematic region where two of these particles carry small momentum [14]. Subleading terms in the emission of two soft scalars computed in Cachazo-He-Yuan (CHY) representation of the amplitude, have been determined for a vast class of theories in Ref. [15]. In the spinor helicity formalism, similar analyses are performed for amplitudes with gravitons and gluons and for scalars of $\mathcal{N} = 4$ super-Yang-Mills in Ref. [16]. New soft theorems in gauge theories with more than one particle are derived in [18] both in four and in any dimensions by using respectively the BCFW and CHY formula.

In this very short paper, which is a summary of the main results obtained in Ref. [1], a purely string approach to the low energy theorems is presented.

Soft gluon and graviton behaviour was also studied in the framework of string theory in Refs. [17–20]. In these

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papers it has been shown that string amplitudes reproduce the soft theorems without any α' correction, α' being the string slope [11].

In this work we consider bosonic string theory and do not only confirm the results obtained in the literature regarding the emission of soft gravitons, but also extend them to the dilaton and Kalb-Ramond fields. In the case of the Kalb-Ramond we do not get any pole term and we find a peculiar relation between $n + 1$ point amplitudes with a soft antisymmetric tensor and n point hard amplitudes, which involve, as an intermediate step, the introduction of holomorphic and antiholomorphic momenta. This handling of the momenta is quite natural in closed string theory, but the relation obtained between amplitudes with and without the soft particle is not a real low energy theorem, because the hard amplitudes are not physical.

The amplitude with a soft graviton and n tachyons is obtained through second subleading order. According to a standard trick, the tachyon is seen as a scalar field with mass $m^2 = -4/\alpha'$ and therefore we have an example of validity of the low energy theorem for massive matter.

String theory is also a powerful tool to get field theory amplitudes. There are few diagrams at each order of the perturbative expansion that are represented as complex integrals on the string moduli space. We have used this compact representation of scattering amplitudes to compute, in bosonic string theory, the colour ordered amplitude with $n + 2$ gluons. On this amplitude two different double soft limits are performed. In one case, contiguous gluons are taken with small momentum, in the other case two soft gluons are separated by a hard particle. In both examples gauge invariant expressions are derived.

2 Single soft limit of string amplitudes

The scattering amplitude involving a massless closed string state, graviton or dilaton, and n closed string tachyons is given by the tensor:

$$M_{\mu\nu} = \frac{8\pi}{\alpha'} \left(\frac{\kappa_d}{2\pi} \right)^{n-1} \int \frac{\prod_{i=1}^n d^2 z_i}{dV_{abc}} \prod_{i < j} |z_i - z_j|^{\alpha' k_i k_j} S_{\mu\nu}, \quad (1)$$

where

$$S_{\mu\nu} = \frac{\alpha'}{2} \int d^2 z \prod_{\ell=1}^n |z - z_\ell|^{\alpha' k_\ell q} \sum_{i=1}^n \frac{k_{i\mu}}{z - z_i} \sum_{j=1}^n \frac{k_{j\nu}}{\bar{z} - \bar{z}_j}. \quad (2)$$

and κ_d is the gravitational coupling constant. The quantities z_i are complex coordinates parametrizing the insertion on the world-sheet of the vertex operators associated to the tachyon states. The coordinate z , without

index, is associated to the massless closed string state. Finally, the soft momentum of massless states is denoted by q .

In principle $M_{\mu\nu}$ describes also the emission of one anti-symmetric tensor from a scattering amplitude with n tachyons. However, this latter contribution vanishes because the world-sheet parity Ω leaves the vertex operators of the tachyon, dilaton, and graviton invariant, while changes the sign of the vertex operator of the Kalb-Ramond.

The main aspect of these new soft theorems consists in finding an operator, \hat{S} , that acting on n -point amplitudes reproduces the soft behaviour of $n + 1$ -point amplitudes. The soft operator is determined by evaluating eq. (2) for small q . Eq. (2) is a sum of integrals on the complex plane that have been explicitly computed in Ref. [1]. Here we only quote the result:

$$\begin{aligned} S_{\mu\nu} = 2\pi \Big\{ & \sum_{i=1}^n k_{i\mu} k_{i\nu} \left[\frac{(\alpha')^2}{2} \sum_{j \neq i} (k_j q) \log^2 |z_i - z_j| \right. \\ & + \frac{1}{k_i q} \left(1 + \alpha' \sum_{j \neq i} (k_j q) \log |z_i - z_j| + \frac{(\alpha')^2}{2} \sum_{j: k \neq i} (k_j q) (k_k q) \right. \\ & \left. \left. \log |z_i - z_j| \log |z_i - z_k| \right) \right] + \sum_{i \neq j} \frac{k_{i\mu} k_{j\nu} + k_{i\nu} k_{j\mu}}{2} \\ & \times \left[-\alpha' \log |z_i - z_j| + \frac{(\alpha')^2}{2} \left(\sum_{k \neq i, j} (k_k q) \left(\log |z_k - z_i| \right. \right. \right. \\ & \left. \left. \left. \log |z_k - z_j| \right) - \sum_{k \neq i} (k_k q) \log |z_i - z_j| \log |z_k - z_i| \right. \right. \\ & \left. \left. - \sum_{k \neq j} (k_k q) \log |z_i - z_j| \log |z_k - z_j| \right) \right] \Big\} + O(q^2). \quad (3) \end{aligned}$$

After a long but straightforward calculation the result of the integrations is rewritten in terms of the differential operators acting on the n -tachyon amplitude:

$$\begin{aligned} \frac{M_{\mu\nu}}{\kappa_d} = & \sum_{i=1}^n \left[\frac{k_{i\mu} k_{i\nu}}{k_i q} - i \frac{k_{i\nu} q^\rho L_{\mu\rho}^{(i)}}{2k_i q} - i \frac{k_{i\mu} q^\rho L_{\nu\rho}^{(i)}}{2k_i q} \right. \\ & - \frac{q^\rho L_{i\mu\rho} q^\sigma L_{i\nu\sigma}}{2k_i q} + \left(\frac{1}{2} (\eta_{\mu\nu} q_\sigma - q_\mu \eta_{\nu\sigma}) - \frac{k_{i\mu} q_\nu q_\sigma}{2k_i q} \right) \frac{\partial}{\partial k_{i\sigma}} \Big] \\ & \times T_n(k_1, \dots, k_n) + O(q^2). \quad (4) \end{aligned}$$

with T_n the n tachyon amplitude defined for example in Ref. [1] and L_i are the angular momentum operators given by:

$$L_i^{\mu\rho} = i \left(k_i^\mu \frac{\partial}{\partial k_{i\rho}} - k_i^\rho \frac{\partial}{\partial k_{i\mu}} \right). \quad (5)$$

The scattering of the dilaton, graviton and Kalb-Ramond is selected by saturating $M_{\mu\nu}$ with the projectors:

$$\text{Graviton } (g_{\mu\nu}) \Rightarrow \epsilon_g^{\mu\nu} = \epsilon_g^{\nu\mu} ; \eta_{\mu\nu} \epsilon_g^{\mu\nu} = 0 \quad (6)$$

$$\text{Dilaton } (\phi) \Rightarrow \epsilon_d^{\mu\nu} = \eta^{\mu\nu} - q^\mu \bar{q}^\nu - q^\nu \bar{q}^\mu \quad (7)$$

$$\text{Kalb-Ramond } (B_{\mu\nu}) \Rightarrow \epsilon_B^{\mu\nu} = -\epsilon_B^{\nu\mu} \quad (8)$$

where \bar{q} is a lightlike vector such that $q \cdot \bar{q} = 1$.

In the case of a graviton, we can neglect the last three terms in the squared bracket of Eq. (4) and we get

$$\epsilon_g^{\mu\nu} \frac{M_{\mu\nu}(q; k_i)}{\kappa_d} = \epsilon_g^{\mu\nu} \sum_{i=1}^n \left[\frac{k_{i\mu} k_{i\nu}}{k_i q} - i \frac{k_{i\nu} q^\rho L_{\mu\rho}^{(i)}}{2k_i q} - i \frac{k_{i\mu} q^\rho L_{\nu\rho}^{(i)}}{2k_i q} - \frac{1}{2} \frac{q_\rho L_i^{\mu\rho} q_\sigma}{k_i q} \right] T_n(k_i) + O(q^2), \quad (9)$$

which, of course, agrees with the soft theorem for the graviton derived in section 3 of Ref. [13].

In the case of the dilaton one gets instead:

$$\epsilon_d^{\mu\nu} \frac{M_{\mu\nu}(q; k_i)}{\kappa_d} = \left[- \sum_{i=1}^n \frac{m_i^2 \left(1 + q^\rho \frac{\partial}{\partial k_i^\rho} + \frac{1}{2} q^\rho q^\sigma \frac{\partial^2}{\partial k_i^\rho \partial k_i^\sigma} \right)}{k_i q} + 2 - \sum_{i=1}^n k_i^\rho \frac{\partial}{\partial k_i^\rho} - \sum_{i=1}^n \left(k_{i\mu} q_\sigma \frac{\partial^2}{\partial k_{i\mu} \partial k_{i\sigma}} - \frac{1}{2} (k_i q) \times \frac{\partial^2}{\partial k_{i\mu} \partial k_{i\mu}} \right) \right] T_n(k_1, \dots, k_n) + O(q^2), \quad (10)$$

where $m_i^2 = -\frac{4}{\alpha'}$ is the squared mass of the closed string tachyon. The dilaton contains terms $O(q^{-1})$ when the other particles are massive, because the three-point amplitude involving a dilaton and two equal particles with mass m is proportional to m^2 .

We have then studied the soft behaviour of amplitudes involving only massless states. In this case the analysis has been done up to the subleading order in the soft expansion and the result is rather complicated but can be written as a convolution integral, $M_{n+1} = S * M_n$, with M_n the amplitude of n massless states in the closed bosonic string, given in ref. [1], and

$$S = 2\pi\epsilon_{q\mu} \bar{\epsilon}_{q\nu} \left(S_{q^{-1}}^{\mu\nu} + S_{q^0}^{\mu\nu} \right) + O(q) \quad (11)$$

with

$$S_{q^{-1}}^{\mu\nu} = \sum_{i=1}^n \frac{k_i^\mu k_i^\nu}{k_i q} \quad (12)$$

and

$$S_{q^0}^{\mu\nu} = \sqrt{\frac{\alpha'}{2}} \sum_{j \neq i} \left[\frac{\sqrt{2\alpha'} k_j^\nu q^\rho}{k_i q} \log |z_i - z_j| \left(k_i^\mu k_{j\rho} - k_{i\rho} k_j^\mu \right) - \left(\frac{\theta_i(\epsilon_i q)}{z_i - z_j} \left(\frac{k_j^\mu k_i^\nu}{k_i q} - \frac{k_j^\mu k_j^\nu}{k_j q} \right) + \text{c.c.} \right) - \left(\frac{\theta_i \epsilon_i^\mu k_j^\nu}{z_i - z_j} + \frac{\bar{\theta}_i \bar{\epsilon}_i^\nu k_j^\mu}{\bar{z}_i - \bar{z}_j} \right) + \frac{k_j q}{k_i q} \left(\frac{\theta_i \epsilon_i^\mu k_i^\nu}{z_i - z_j} + \frac{\bar{\theta}_i \bar{\epsilon}_i^\nu k_i^\mu}{\bar{z}_i - \bar{z}_j} \right) \right] - \sum_{j \neq i} \frac{(\theta_j \epsilon_j q)(\theta_i \epsilon_i^\mu)}{(z_i - z_j)^2} \times \left(\frac{k_j^\nu}{k_j q} - \frac{k_i^\nu}{k_i q} \right) - \sum_{j \neq i} \frac{(\bar{\theta}_j \bar{\epsilon}_j q)(\bar{\theta}_i \bar{\epsilon}_i^\nu)}{(\bar{z}_i - \bar{z}_j)^2} \left(\frac{k_j^\mu}{k_j q} - \frac{k_i^\mu}{k_i q} \right). \quad (13)$$

The polarizations of the massless states are conveniently written in the form $\epsilon_{\mu\nu} = \theta \epsilon_\mu \bar{\theta} \bar{\epsilon}_\nu$, with $(\theta, \bar{\theta})$ Grassmanian variables and c.c. denotes the complex conjugate.

If we use the polarization of a graviton, given in Eq. (6), we get the soft behavior for a graviton in agreement with the result of Ref. [13]. In the case of the dilaton we get instead:

$$\frac{M_{n+1}}{\kappa_d} = \left[2 - \sum_{i=1}^n k_{i\mu} \frac{\partial}{\partial k_{i\mu}} \right] M_n + O(q), \quad (14)$$

in agreement with the result of Ref. [17]. We have checked that the previous soft behaviour is also obtained in the case of the superstring.

In order to define a low energy theorem for the antisymmetric tensor, it is convenient to keep distinct the holomorphic, k_i , and anti-holomorphic, \bar{k}_i , momentum coming from the factorized structure of the vertices in closed string theory. According to such a separation one gets:

$$\frac{i M_{n+1}}{\kappa_d} = \epsilon_{q\mu\nu} \sum_{i=1}^n \left[\frac{(L_i - \bar{L}_i)^{\mu\nu}}{2} + \frac{k_i^\nu q_\rho (S_i - \bar{S}_i)^{\mu\rho}}{k_i q} \right] M_n \Big|_{k=\bar{k}} \quad (15)$$

with

$$S_i^{\mu\nu} = i \left(\epsilon_i^\mu \frac{\partial}{\partial \epsilon_{i\nu}} - \epsilon_i^\nu \frac{\partial}{\partial \epsilon_{i\mu}} \right), \quad \bar{S}_i^{\mu\nu} = i \left(\bar{\epsilon}_i^\mu \frac{\partial}{\partial \bar{\epsilon}_{i\nu}} - \bar{\epsilon}_i^\nu \frac{\partial}{\partial \bar{\epsilon}_{i\mu}} \right) \quad (16)$$

and \bar{L} is the angular momentum operator written in terms of the anti-holomorphic momenta \bar{k} . Eq. (15) reproduces the soft behavior of the antisymmetric tensor, but it is not a real soft theorem as in the case of the graviton and dilaton because, due to the separation of k and \bar{k} , the amplitude M_n is not a physical amplitude.

3 Double soft limit of string amplitudes

In this section we consider the color-ordered scattering amplitude, $M_{2g;ng}$, involving $(n+2)$ gauge fields living

on the world-volume of a Dp -brane of the bosonic string and we compute the leading double-soft behavior when two contiguous gluons become simultaneously soft.

We denote with (ϵ_{q_1}, q_1) and (ϵ_{q_2}, q_2) the polarizations and momenta of two contiguous gluons that eventually will become soft and with (ϵ_i, k_i) the polarizations and momenta of the remaining gluons. The amplitude has been computed in detail in ref. [1], and here we give only the result written as a convolution integral, $M_{2g;ng} = M_{ng} * G_n$, between the n gluon amplitude and the quantity G_n which contains all the information about double soft behaviour of the gluons

$$G_n = 2\alpha' g_{p+1}^2 \int_0^{z_{n-1}} dw_1 \int_0^{w_1} dw_2 (w_1 - w_2)^{2\alpha' q_1 q_2} \times \prod_{i=1}^n \prod_{a=1}^2 \left[(z_i - w_a)^{2\alpha' k_i q_a} e^{\sqrt{2\alpha'} \frac{\theta_i \epsilon_i q_a}{z_i - w_a}} \right] \left\{ \frac{(\epsilon_{q_1} \epsilon_{q_2})}{(w_1 - w_2)^2} + \left[\sum_{i=1}^n \frac{\theta_i (\epsilon_i \epsilon_{q_1})}{(z_i - w_1)^2} - \sum_{i=1}^n \frac{\sqrt{2\alpha'} (k_i \epsilon_{q_1})}{z_i - w_1} + \frac{\sqrt{2\alpha'} (\epsilon_{q_1} q_2)}{w_1 - w_2} \right] \times \left[\sum_{j=1}^n \frac{\theta_j (\epsilon_j \epsilon_{q_2})}{(z_j - w_2)^2} - \sum_{j=1}^n \frac{\sqrt{2\alpha'} (k_j \epsilon_{q_2})}{z_j - w_2} - \frac{\sqrt{2\alpha'} (\epsilon_{q_2} q_1)}{w_1 - w_2} \right] \right\}.$$

The latter integral has been computed in the limit of small momenta q_1, q_2 and the resulting expression turns out to be:

$$G_n = \frac{g_{p+1}^2}{q_1 q_2} \left\{ \left[- \frac{(\epsilon_{q_1} \epsilon_{q_2}) k_n (q_2 - q_1) + q_1 q_2}{2s_n} + \frac{(\epsilon_{q_1} q_2)(\epsilon_{q_2} k_n) - (\epsilon_{q_2} q_1)(\epsilon_{q_1} k_n)}{s_n} + \frac{(\epsilon_{q_1} k_n)(\epsilon_{q_2} k_n)(q_1 q_2)}{(k_n q_2) s_n} + k_n \leftrightarrow k_{n-1} \right] - \frac{(\epsilon_{q_1} k_{n-1})(\epsilon_{q_2} k_n)(q_1 q_2)}{(k_{n-1} q_1)(k_n q_2)} \right\}, \quad (17)$$

where $s_\alpha = k_\alpha (q_1 + q_2) + q_1 q_2$ with $\alpha = n, n-1$. Eq.(17) is gauge invariant and behaves as $\frac{1}{q_1 q_2}$ in the double-soft limit, i.e. when both q_1 and q_2 simultaneously go to zero.

The double-soft behaviour of the $n+2$ -point color ordered amplitude with two soft particles separated by a hard one is evaluated along the same lines of the contiguous case. The resulting expression is again a convolution between the n point gluon amplitude and the momenta dependent quantity:

$$G_{2g} = g_{p+1}^2 \left[\frac{k_{n-2} \epsilon_{q_1}}{k_{n-2} q_1} \left(\frac{k_{n-1} \epsilon_{q_2}}{k_{n-1} q_2} - \frac{k_n \epsilon_{q_2}}{k_n q_2} \right) + \frac{k_{n-1} \epsilon_{q_1}}{k_{n-1} q_1} \frac{k_n \epsilon_{q_2}}{k_n q_2} - \frac{(\epsilon_{q_1} k_{n-1})(\epsilon_{q_2} k_{n-1})}{k_{n-1} (q_1 + q_2) + q_1 q_2} \left(\frac{1}{k_{n-1} q_1} + \frac{1}{k_{n-1} q_2} \right) \right] \quad (18)$$

It is easy to check that the soft factor is gauge invariant up to terms of order $q_{1,2}^0$ as in the case of two contiguous soft gluons.

4 Concluding Remarks

We have presented the results of Ref. [1], showing that bosonic string theory is a useful framework for computing low-energy properties of scattering amplitudes in a gauge covariant way and for any dimensions. The framework also allows to extend the results straightforwardly to higher orders in soft-momenta and to directly apply it to superstrings, which we plan to present in a future work.

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